STEREOSCOPIC PARALLAX

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PARALLAX

• Apparent shift in the position of an object, with respect to a frame of reference, caused by a shift in the position of observation
• Change in position of an image from one photo to the next is caused by aircraft’s motion
  – Called stereoscopic parallax, x parallax, or simply parallax
• Two important aspects of stereoscopic parallax
  – Parallax of any point is directly related to the elevation of the point
  – Parallax is greater for high points than for low points
STEREOSCOPIC PARALLAX

$P_a = x_a - x'_a$

STEREOSCOPIC PARALLAX

- Parallax measured in flight-line axis system

Flight line for photos 1 and 2

Flight line for photos 2 and 3
**MONOSCOPIC PARALLAX MEASUREMENT**

- Mark conjugate principal points
- Align flight line axis
- Parallax: \( p_b = D - d_b \)

**PRINCIPLE OF FLOATING MARK**

- When viewing in stereo, 2 small identical marks etched on clear glass
  - Called half marks
- Half marks shifted until they fuse into single mark
- If marks moved closer together, they appear to rise
- If moved apart, marks appear to fall
- Spacing of half marks, hence their parallax, varied so floating marks appears to rest exactly on terrain
PARALLAX OF PRINCIPAL POINT

- Parallax of left ground principal point is photo base $b'$ measured on right photo, and the parallax of right ground principal point is photo base $b$ measured on left photo.
- For moderate relief, $b \approx b'$
- Photo base is average of two values

STEREOSCOPIC PARALLAX MEASUREMENT
**STEREOSCOPIC PARALLAX MEASUREMENT**

- Parallax bar measurement
  \[ p_a = x_a - x'_a = D - (K - r_a) = (D - K) + r_a \]
- Substituting parallax bar constant \( C \)
  \[ p_a = C + r_a \]
- To compute \( C \), measure parallax monoscopically and take micrometer reading
  \[ C = p - r \]

**DEVELOPMENT OF PARALLAX EQUATIONS**

![Diagram of stereoscopic parallax measurement](image)
DEVELOPMENT OF PARALLAX EQUATIONS

- Triangles LOW and Low, write scale:

\[ \frac{Lo}{LO} = \frac{Lw}{LW} = \frac{ow}{OW} \Rightarrow \frac{f}{H-h} = \frac{x}{X} \]

- From triangles Lwp and LWP, write scale:

\[ \frac{Lw}{LW} = \frac{wp}{WP} = \frac{f}{H-h} = \frac{y}{Y} \]

DEVELOPMENT OF PARALLAX EQUATIONS

- Using triangles L'O'W, L'o'w', L'w'p', L'WP

\[ \frac{L'o'}{L'O'} = \frac{L'w'}{L'W} = \frac{w'p'}{WP} = \frac{y'}{Y} \Rightarrow y' = \frac{f}{Y} \frac{H-h}{H} \]

- From last two relationships:

\[ \frac{y}{Y} = \frac{f}{H-h} \quad \frac{y'}{Y} = \frac{f}{H-h} \]

- Yielding:

\[ y = y' \]
DEVELOPMENT OF PARALLAX EQUATIONS

- In triangles $\triangle LWL'$ & $\triangle Lww'$
  - $LL'$ is parallel to $ww'$
  - $LW$ is parallel to $Lw$
  - $L'W$ is parallel to $Lw'$
  - The two triangles are similar triangles
  - Corresponding altitudes are $(H - h)$ and $f$

DEVELOPMENT OF PARALLAX EQUATIONS

- From similar triangles
  \[
  \frac{f}{H - h} = \frac{ww'}{B}
  \]

- Since $ww' = x - x' = p$

- then
  \[
  \frac{f}{H - h} = \frac{p}{B}
  \]
DEVELOPMENT OF PARALLAX EQUATIONS

- The parallax equations are:

\[
H - h = \frac{B}{p} f \Rightarrow h = H - \frac{Bf}{p}
\]

\[
X = \frac{B}{p} x
\]

\[
Y = \frac{B}{p} y
\]

PARALLAX EQUATIONS

- Valid for
  - Truly vertical photographs only
  - Photos taken from same flying height
  - Coordinates \((x, y, x', y')\) related to flight line axis system

- Ground coordinates not related to true ground coordinates but to the coordinate system of the stereopair
ELEVATION BY PARALLAX DIFFERENCES

- Recall parallax formula:
  \[ h = H - \frac{Bf}{p} \]

- Rearrange for points a and c
  \[ p_c = \frac{fB}{H-h_c} \quad p_a = \frac{fB}{H-h_A} \]
**ELEVATION BY PARALLAX DIFFERENCES**

Parallax difference: \[ \Delta p = p_a - p_c \]

\[
\frac{fB}{H - h_A} - \frac{fB}{H - h_C} = \frac{fB (H - h_C) - fB (H - h_A)}{(H - h_A)(H - h_C)} = \frac{fB (h_A - h_C)}{(H - h_A)(H - h_C)}
\]

**ELEVATION BY PARALLAX DIFFERENCES**

- Substituting parallax formula for flying height above the terrain

\[ \Delta p = \frac{fB(h_A - h_C)}{(fB/p_a)(H - h_C)} = \frac{p_a(h_A - h_C)}{H - h_c} \]

- From which \[ h_A = h_C + \frac{\Delta p (H - h_C)}{p_a} \]
ELEVATION BY PARALLAX DIFFERENCES

• Alternative development

\[ \Delta h = h_A - h_C \]

\[ = \left( H - \frac{Bf}{p_a} \right) - \left( H - \frac{Bf}{p_c} \right) \]

\[ = \frac{Bf \Delta p}{p_c (p_c + \Delta p)} \]

ELEVATION BY PARALLAX DIFFERENCES

• Since ground principal points lie on same datum

\[ o_1 o_2 = o'_1 o'_2 = b \]

• Since \( O_1, O_2 \) and C lie at same elevation, their parallaxes are the same

\[ p_c = b \]
ELEVATION BY PARALLAX DIFFERENCES

• From figure

\[ \frac{b}{B} = \frac{f}{H - h_c} \]

• From which

\[ B = \frac{(H - h_c)b}{f} \]

ELEVATION BY PARALLAX DIFFERENCES

• Substitute in elevation difference formula and recognizing that \( p_c = b \)

\[ \Delta h = \frac{(H - h_c)\Delta p}{b + \Delta p} \]
ERROR EVALUATION

- Some sources of errors
  - Locating and marking flight lines
  - Orienting stereopairs for parallax measurements
  - Parallax and photo coordinate measurements
  - Shrinkage or expansion of photos
  - Unequal flying heights
  - Tilted photographs
  - Errors in ground control
  - Other errors: camera lens distortion, atmospheric refraction distortion

ERROR EVALUATION

- General approach – differentiate equation
- Example for basic parallax equations

\[ h = H - \frac{Bf}{p} \]

\[ \frac{\partial h}{\partial H} = 1 \ ; \quad \frac{\partial h}{\partial B} = -\frac{f}{p} \ ; \quad \frac{\partial h}{\partial p} = \frac{Bf}{p^2} \]